Probabilistic Model of Mean Stress Effects in Strain-Life Fatigue

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Abstract

This paper proposes a modification of the Walker model to account for the mean stress effect on fatigue life of structures and the associated scatter of experimental data. To account for various sources of uncertainty, a framework which permits dealing with run-outs and provides an analytical probabilistic definition of the whole strain-life field, both in the low-cycle and high-cycle fatigue regions is used. The proposed model is an improvement relative to the Smith-Watson-Topper model for materials that are significantly more or less sensitive to mean stress. Model’s applicability is shown for various metals.

Key words: Fatigue life; mean stress effects; Walker equation; strain-life curve; Smith-Watson-Topper equation; Weibull regression.

1. Introduction

Constant amplitude loading of structures or components in the presence of mean stress/strain can result in failures which occur sooner or later than would be estimated by commonly used life prediction models. This happens because the mean stress has a significant influence of fatigue life: tensile mean stress is detrimental to the fatigue life, while compressive mean stress is beneficial [1]. Therefore, fatigue life estimates using either stress-life or strain-life approaches necessarily include the effect of mean stress. Various methods have been developed to model mean stress effects on the fatigue behaviour of metals [2–9].

This paper discusses a method of incorporating the experimental data scatter of mean stress into strain-life equations, therefore a number of sets of test data are compared with strain-life equations generalized using a Walker mean stress-like method. Emphasis is placed on the Walker method, as previous work in a stress-life context [10,11] and strain-life context [12] has shown that it is superior to other common methods of handling mean stress effects. Previous work [12] shows that the Walker mean stress equation gives excellent results for steels, titanium and aluminium alloys. In addition, Walker’s approach has a number of advantages: (1) all data at all mean stresses can be combined into a single fitting procedure to determine the constants for the stress-life curve, (2) the Walker parameter γ that arises

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from this fitting is related to the sensitivity of the material to mean stress, giving this approach a versatility that is not possessed by the other common mean stress methods.

**Nomenclature**

- $b_w$: exponent constant for a Walker method stress-life fit
- $c_w$: exponent constant for a plastic strain versus Walker-equivalent-life curve
- $N_f$: fatigue life; cycles to failure
- $N^*_w$: Walker equivalent life
- $R$: stress ratio, $R = \sigma_{\text{min}}/\sigma_{\text{max}}$
- $R_{\varepsilon}$: strain ratio, $R_{\varepsilon} = \varepsilon_{\text{min}}/\varepsilon_{\text{max}}$
- $\varepsilon_a$: strain amplitude
- $\Delta \varepsilon$: strain range, $\Delta \varepsilon = 2\varepsilon_a$
- $\varepsilon_m$: mean strain
- $\varepsilon_{pa}$: plastic strain amplitude
- $\gamma$: fitting constant for the classical Walker equation
- $\hat{\gamma}$: fitting constant for the present model
- $\sigma_{\text{max}}$: maximum stress
- $\sigma_{\text{min}}$: minimum stress
- $\sigma_a$: stress amplitude
- $\sigma_m$: mean stress
- $\Delta \sigma$: stress range, $\Delta \sigma = 2\sigma_a$

The fatigue lives of similar specimens or structures under the same fatigue load can be significantly different due to the variability of parameters such as material properties and microstructure (e.g. large inclusions or defects) or geometrical features of a component. This scatter of fatigue-lives is often the reason for differences between deterministically predicted fatigue-lives and those observed in service. Therefore, in order to capture and include the experimental scatter in the fatigue-life assessment procedures, a probabilistic approach is needed. The standard practice of statistically analyzing stress/strain-life data [13] gives only an elementary procedure that assumes that the size and the shape of life distributions are similar at all strain levels. In other words, the distribution is assumed to be insensitive to the strain/stress level. However, our previous work [14] shows that the scatter band is narrower at high stress amplitudes ($\sigma_a$) and larger at low stress amplitudes. At a high $\sigma_a$-value (Low Cycle (LCF) to High Cycle Fatigue (HCF)), surface condition are less important for crack nucleation because microcracks are initiated early in the fatigue life. These microcracks are followed immediately by further crack growth. As a result, scatter will be relatively low. However, at a low $\sigma_a$-value (Very High Cycle Fatigue (VHCF)), crack nucleation and the first microcrack growth meet structural barriers in the material. Nucleation can be dependent on local surface inhomogeneities, small surface irregularities or even slight surface damage. These surface conditions can vary from specimen to specimen, and have a significant effect on the duration of the initiation period. As a result, more scatter is found at high endurance. Therefore a reliable probabilistic model needs to include this difference between LCF and HCF in a unified approach.

Different methods have been developed for solving cyclic loading fatigue reliability problems [14–18]. Among them, the probabilistic framework of Castillo [20] provides a very appealing model as it: (a) provides an analytical probabilistic definition of the whole strain-life field as quantile curves, both in the low-cycle and high-cycle fatigue regions, (b) deals directly with the total strain, without the need of separating its elastic and plastic strain components, (c) gives explicitly the probabilistic strain-life ($P - \varepsilon - N$) field, (d) permits dealing with run-outs and with specimens of different lengths. Therefore, this paper extends Castillo’s model to include a generalization of Walker mean stress method and shows its applicability to various metals.
2. Strain-life with mean stress effect

2.1. Classical Walker model

A short overview of the traditional Walker equation and the associated notations are summarized here for a complete presentation. As illustrated in Figure 1, for a cyclically varying stress, \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) are the maximum and minimum stresses, respectively, \( \sigma_a \) is the stress amplitude, \( \sigma_m \) is the mean stress, \( \Delta \sigma = 2\sigma_a \) is the stress range, \( R = \sigma_{\text{min}}/\sigma_{\text{max}} \) is the stress ratio and:

\[
\sigma_a = \frac{\sigma_{\text{max}}}{2}(1-R), \quad \sigma_m = \frac{\sigma_{\text{max}}}{2}(1+R)
\]  

(1)

Similar definitions are applied to strain: \( \varepsilon_a \) is the strain amplitude, \( \Delta \varepsilon = 2\varepsilon_a \) is the strain range, \( \varepsilon_m \) is the mean strain, and \( R_\varepsilon = \varepsilon_{\text{min}}/\varepsilon_{\text{max}} \) is the strain ratio.

\[ \text{Fig. 1: Constant amplitude cyclic stressing and definitions of stress variables.} \]

Strain amplitude versus Walker equivalent life curve is given by [12]:

\[
\varepsilon_a = \frac{\sigma'_{fw}}{E}(2N^*_w)^{b_w} + \varepsilon'_{fw}(2N^*_w)^{c_w}
\]  

(2)

where the Walker equivalent life is:

\[
N^*_w = N_f \left( \frac{1}{2} \right)^{(1-\gamma)/b_w}
\]  

(3)

For a better comparison with the current model, these equations are re-written as:

\[
\varepsilon_a \left( \frac{2}{1-R} \right)^{(1-\gamma)} = \frac{\sigma'_{fw}}{E}(2N_f)^{b_w} + \varepsilon'_{fw}(2N_f)^{c_w} \left( \frac{1}{2} \right)^{(e_w/b_w-1)(1-\gamma)}
\]  

(4)

Previous work [12] presents a detailed methodology to determine the above parameters: the constants \( \sigma'_{fw}, b_w \) and \( \gamma \) are obtained first by stress-life data fitting as the first term of equation (2) corresponds to the elastic strain. The second term of equation (2) corresponds to the plastic strain amplitude given by: \( \varepsilon_{pa} = \varepsilon'_{fw}(2N^*_w)^{c_w} \) therefore a linear-least-squares fit for \( \log(2N^*_w) - \log \varepsilon_{pa} \) is performed in order to determine constants \( \varepsilon'_{fw} \) and \( c_w \).

2.2. Probabilistic Walker model

The probabilistic strain-life Weibull regression model proposed by Castillo and his collaborators [19–21] can be written in a compact form as:

\[
f(\varepsilon_a)f(N_f) = C
\]  

(5)

where function \( f \) has a logarithmic form

\[
f(x) = \log x - \alpha_i = \log \left( \frac{x}{e^\alpha} \right), \quad i = 1, 2.
\]  

(6)
The two constants \( \alpha \) have clear meaning: \( \alpha_1 \) (for \( x = x_k \)) represents a threshold value of strain amplitude (i.e. limit for infinite number of cycles to failure) whereas \( \alpha_2 \) (for \( x = N_f \)) is the threshold value of life (i.e. number of cycles for which the strain is theoretically infinite). The two constants can be determined using a constrained least-square method and experimental data. After this first step, the set of values

\[
C_k = f(\varepsilon_{ak})f(N_{fk}), \quad k = 1, ..., T \text{(number of tests)}
\]

(7)
corresponding to experimental data couples \((N_{fk}, \varepsilon_{ak})\), is fitted with a three-parameter Weibull distribution:

\[
C_k \sim W(\lambda, \delta, \beta)
\]

(8)

For the determination of parameters of this distribution (\( \lambda \) is the parameter defining the position of the corresponding zero-percentile hyperbola, \( \delta \) corresponds to the scale factor, and \( \beta \) is the Weibull shape parameter of the CDF), the methods of maximum likelihood or probability weighted moments can be used [22].

Is is noted here that the selection of Weibull distribution for variable \( C_k \) is not random but it is based on statistical conditions that a \( \varepsilon_a - N_f \) curve should satisfy as detailed in [19]. These conditions are: (a) weakest link principle, (b) stability, (c) limit behavior, (d) limited range, and (e) compatibility. Among them, the latter condition requires that the acceptable solution, the 3-parameter Weibull distribution.

In order to take into account the effects of stress ratio, the above model was modified as [23]:

\[
f(SWT^*)f(N_f) = C
\]

(9)

where the Smith-Waton-Topper field is \( SWT^* = \sigma_{max} \varepsilon_a \).

In the present work, the above model is generalized using a Walker mean stress-like equation. Emphasis is placed on the Walker method, as previous work [10–12] has shown that it is superior to other common methods of handling mean stress effects. It was also shown that different metals have different sensitivities to mean-stress effects; among the metals tested, higher strength steels have the highest sensitivity. The model can be described in a general form as:

\[
f(\varepsilon^{\ast}_a)f(N_f) = C
\]

(10)

where the Walker-like strain field is:

\[
\varepsilon^{\ast}_a = \varepsilon_a \left( \frac{2}{1 - R} \right)^{1 - \hat{\gamma}}
\]

(11)

and where \( R \) is the stress ratio \( R = \sigma_{min}/\sigma_{max} \). The Walker-like parameter \( \hat{\gamma} \) that arises from this fitting is related to the sensitivity of the material to mean stress, giving this approach a versatility that is not possessed by the other common mean stress methods. As above, the function \( f \) has a logarithmic expression \( f(x) = \log x - \alpha \) and the model parameters \( \alpha_1, \alpha_2, \hat{\gamma}, \lambda, \delta, \beta \) are estimated using available methods fitted with experimental data.

Explicitly the above equation can be written as percentile curves as:

\[
\varepsilon_a \left( \frac{2}{1 - R} \right)^{1 - \hat{\gamma}} = \exp \left\{ \alpha_1 + \frac{\lambda + \delta [- \log(1 - p)]^{1/\beta}}{\log N_f - \alpha_2} \right\}
\]

(12)

where \( p \in [0, 1] \) is the probability level. It is noted here that the present model adds an extra parameter, \( \hat{\gamma} \), which along \( \alpha_i \) can be determined using a constrained least-square method:

\[
\min_{\alpha_1, \alpha_2, \gamma, K} \sum_{k=1}^{T} w_k \left( \log N_{fk} - \alpha_2 - \frac{K}{\log \varepsilon_{ak} + (1 - \hat{\gamma}) \log \left( \frac{2}{1 - R_k} \right) - \alpha_1} \right)^2
\]

(13)

subjected to:

\[
\begin{align*}
\alpha_2 & \leq \min_k \log N_{fk} \\
\log \varepsilon_{ak} + (1 - \hat{\gamma}) \log \left( \frac{2}{1 - R_k} \right) - \alpha_1 & > 0
\end{align*}
\]

(14)
The Weibull parameters $\lambda, \delta, \beta$ are determined using the methods of maximum likelihood or probability weighted moments [22].

A comparison of classical model (eq. (4)) and current model (eq. (12)) explains the difference between them. Both are based of five parameters and the left side of these equations are the same. Nevertheless, the classical model separates elastic and plastic data and their corresponding $\log \varepsilon_{e/pa} - \log N_f$ variations are linear, whereas the current model deals with total strain and the corresponding $\log \varepsilon - \log N_f$ variation is non-linear. Because these two equations are different, $\gamma$ and $\hat{\gamma}$ are similar yet different (in other words, for a given data set, $\gamma$ and $\hat{\gamma}$ cannot be compared).

3. Results

3.1. Experimental data

Experimental data with emphasis on 7075-T651 aluminium alloy collected from the literature and supplemented with our own data [32] is used here. A short description of the data and their original sources are given below, followed by the comparison with the current model (Figure 2). 

Al 7075-T651. Because of its superior strength, 7075-T651 is heavily utilized by the aircraft industries. Due to this wide application throughout aircraft and aerospace structures, our team previous’ work [32,36] focused on this particular alloy and generated long-life data to support this effort. As described below, fatigue data were collected from several sources and compiled into a single data set:

Endo and Morrow [24] included fatigue tests on 7075-T6 aluminum for a 1/4 in diameter specimen with uniform gauge section of 3/4 in and for a hourglass specimen to avoid buckling for larger strain amplitudes. The range of cycles to failure for this data set goes from 10 to 10,000.

Howell and Miller [25] included typical values of fatigue strength at different number of cycles to failure (10,000 to $5 \times 10^8$) for varying $R$-ratios. Representative values at $R = 0$ were obtained from the reference and used in this analysis. Data are obtained from axial tests on standard round specimens.

Gregor and Grossman [28] - the material was a 1/2 in diameter rod and the specimens were of round constant gauge section type. Fatigue tests were performed using a rotating bending apparatus working at 10,000 rpm.

Stanzl-Tschegg et. al. [32] - the material was in plate form with a thickness of 20 mm and the test specimens were of constant cross section type, however specifically designed to be tested with ultrasonic testing equipment, therefore such that one of their natural frequencies was around 20kHz, and with a diameter of 4mm. Two $R$-ratios were tested: $R = -1$ and $R = 0.5$.

Team’s unpublished data - Data from a complementary effort lead by the present authors that involved the testing of 7075-T651 3/4 in thick plate were also included in the set. Specimens were of cylindrical constant cross section type and tested at $R = -1$.

Lazan [26,27] included tests on 7075-T6 aluminum in rolled bar and extrusion form. However, analysis of the data set for rolled bar material revealed very unusual behavior and it is not used in the current analysis. This has been noted by the author as well: “the fatigue strength of the rolled 75S-T6 tested in that program was abnormally low”. The specimens tested were of hourglass type and fatigue lives were in the range 10,000 to $10^6$.

Other sources [29–31] were also considered, however the data were either from specific rod forms that may not be representative of the material in this work or were obtained from initially overstrained tests, therefore showing significant deviation from constant amplitude data, especially in the long life region.

Metallic alloys. Due to space limitation, among the 18 sets of data previously analysed [12], only 5 are illustrated below. These are selected to belong to different metals (steel, titanium and aluminum) with $10^7 - 10^7$ cycles to failure. A short summary of the data is given in Table 1.
Table 1: Range of cyclic data and Walker parameter $\gamma$.

<table>
<thead>
<tr>
<th>Material</th>
<th>Source (from Reference list)</th>
<th>Range of data</th>
<th>Walker parameter, $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17MnCrMo33 St</td>
<td>[33]</td>
<td>23–2840000</td>
<td>0.686</td>
</tr>
<tr>
<td>AISI 4340 St</td>
<td>[34]</td>
<td>222–901430</td>
<td>0.590</td>
</tr>
<tr>
<td>SAE 1045 St, 705 HB</td>
<td>[35]</td>
<td>2.5–161250</td>
<td>0.494</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>[33]</td>
<td>214–1100000</td>
<td>0.561</td>
</tr>
<tr>
<td>2024-T3 Al</td>
<td>[33]</td>
<td>77–2700000</td>
<td>0.600</td>
</tr>
<tr>
<td>7075-T651 Al</td>
<td>[24–28,32]</td>
<td>8–3470000000</td>
<td>0.558</td>
</tr>
</tbody>
</table>

3.2. Comparison with test data

As described by the equation (12), Figure 2 shows the variation of the Walker-modified strain amplitude $e^*_a$ with the fatigue life, $N_f$ for five values of probability: $p = 1, 10, 50, 90, 99%$. Experimental data, included in the figures is used to find the five unknowns of Equation (12). These figures prove that the presented model correlates with the associated data well, as indicated by most data points located in between 1% and 99% curves. Although not included in this paper, very similar conclusions are obtained for the 13 remaining data sets previously analysed [12].

It is noted here that the Al 7075-T651 data set (Figure 2(f)) includes run-outs (tests which are interrupted before the specimens fail) and these are transformed using the likelihood function for the right censored sampling scheme. The original data is shown with red triangles whereas the transformed data used for the fit is shown with black squares.

Each figure and Table 1 list the values for the Walker-like parameter, $\gamma$ which varies from 0.494 for SAE 1045 steel (705 HB) to 0.686 for 17MnCrMo33 steel. This variation illustrates the importance of considering $\gamma$ as a parameter and not as constant as given by the Smith-Watson-Topper model.

4. Conclusions and future work

This paper presents a novel model to incorporate the Walker mean stress equation into the strain-life curve which gives excellent results for various alloys (steel, titanium and aluminium) and incorporates a probabilistic framework to deal with the experimental scatter. The model improves upon the framework developed by Castillo [20], by including an additional parameter $\gamma$ which is related to the sensitivity of the material to mean stress. This addition gives the current model a versatility that is not possessed by the other common mean stress methods.

Parallel work [36] is conducted to estimate life due to variable amplitude loading using an approach that incorporates this Walker mean stress correction method into the strain-life curve. Future work will continue the current improvements by including confidence levels into the model. Future work will build a similar framework for the stress-strain equation which is also important for structural fatigue life estimations.

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Fig. 2: Walker-strain amplitude versus fatigue life for six metallic alloys.