ANALYSIS OF ROTOR LOAD HARMONIC COMPOSITION AND LOAD TRANSFERRENC BETWEEN FRAMES

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ABSTRACT

Loads prediction for helicopters is more complex than that for fixed wing aircraft given that the aerodynamic forces that produce the air vehicle’s lift and moments are generated by rotating components (e.g., the main and tail rotors). This paper examines rotor load content as a function of harmonics of the rotor angular speed. Accurate rotor loads prediction is driven by a clear understanding of the underlying physics of the problem. This understanding is important for several reasons. First, additive effects of multiple load components fed from the rotating frame to the fixed frame influence the vibratory response of the hub and fuselage. Second, structural fatigue life is driven by load magnitude as well as the number of applications of each load. The mechanics of load transference between the rotating and fixed frames will also be examined. Examples for 4-bladed and 7-bladed rotors will be presented. Definitions of cyclic/whirl (moment transference), collective (force transference), and reactionless/warp (zero force, zero moment transference) modes will be presented for a generic n-bladed rotor. US Army UH-60A Black Hawk flight test data will be examined. Blade lift, main rotor shaft bending, pushrod axial load, and fixed system servo loads will be derived and compared with measured UH-60A results. The intent of this paper is to provide clear insight into the nature of rotor load harmonic composition and load transference between the fixed and rotating frames for a rotor - two areas not well documented (and never visually described) in the literature.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>AS</td>
<td>UH-60A aft servo load, lb (N).</td>
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<td>FS</td>
<td>UH-60A forward servo load, lb (N).</td>
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<tr>
<td>iP</td>
<td>i^-th per-rev response content (1P, 2P, ...).</td>
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<tr>
<td>LS</td>
<td>UH-60A lateral servo load, lb (N).</td>
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<td>nb</td>
<td>Number of blades.</td>
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<td>Pi</td>
<td>UH-60A pushrod (pitch link) axial load for the i^-th blade (Fx; lb (N)).</td>
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<td>r</td>
<td>Rotor blade radial station, ft (m).</td>
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<td>R</td>
<td>Rotor bade length, ft (m).</td>
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<td>µ</td>
<td>Advance ratio (air vehicle forward velocity normalized by main rotor blade tip speed; V/ωR).</td>
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<tr>
<td>Ω</td>
<td>Rotor angular speed; positive counter-clockwise (rpm, Hz, or rad/s).</td>
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<tr>
<td>ψ</td>
<td>Rotor azimuth; zero for blade pointing aft; positive counter-clockwise (deg).</td>
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INTRODUCTION

Loads prediction for helicopters is more complex than that for fixed wing aircraft given that the aerodynamic forces that produce the air vehicle’s lift and moments are generated by rotating components (e.g., the main and tail rotors). This paper examines rotor load content as a function of harmonics of the rotor angular speed. Accurate rotor loads prediction is driven by a clear understanding of the underlying physics of the problem. This understanding is important for several reasons. First, add-
additive effects of multiple load components fed from the rotating frame (rotor) to the fixed frame (fuselage) influence the vibratory response of the hub and fuselage. For example, on the fixed side, pilot and automatic control system inputs (e.g., stick and pedal inputs to control air vehicle orientation and velocity) are fed via control servos to the stationary swashplate of the main rotor. A rotating swashplate (rotating relative to the stationary swashplate) transfers these inputs from the fixed system control servos to pushrods in the rotating frame. Each pushrod is attached at one end to the rotating swashplate and to a rotor blade at the other end. The pushrods, in turn, set the pitch and flap angles of each corresponding blade, thus setting the lift (magnitude and direction) of each blade. The rotor thrust is a summation of the lift for all blades. See Figure 1 for a conceptual drawing of a helicopter main rotor system.

Table 1 lists the set of UH-60A flights used for analysis herein (flights 84 and 85, totaling 51 flight counters [2, 5]). This flight numbering is used to match the numbering used in the test program (e.g., “flight 84”). A flight counter (e.g., “c8534” - flight point 34 from flight 84) is a discrete test point within a test flight; a specified flight maneuver or period of steady flight. Each flight counter is typically 5 to 45 seconds in duration, based on the degree of steadiness of the response with time.

TABLE 1. UH-60A FLIGHTS, FLIGHT COUNTERS USED IN ANALYSIS.

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<tr>
<th>Flight number</th>
<th>Flight counters</th>
<th>Objective</th>
<th>Description</th>
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<tr>
<td>84</td>
<td>22</td>
<td>Steady and maneuvering airloads</td>
<td>Level flight; accel/decel; hover</td>
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<tr>
<td>85</td>
<td>29</td>
<td>Steady and maneuvering airloads</td>
<td>Level flight; steady turns; roll reversals</td>
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<tr>
<td>Total:</td>
<td>51</td>
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measured on one blade (hereafter blade 1) using 221 pressure transducers installed in nine radial arrays. Additionally, 21 transducers were installed in blade 3 (180 deg lag relative to blade 1) at nine radial stations to measure blade bending along the three axes: flapwise (normal) bending, edgewise (in-plane) bending, and torsional moment. Main rotor shaft bending, pushrod (pitch link) axial loads, control servo loads, as well as a range of other responses were measured as well. A total of 31 data flights were flown, totaling 57 flight hours.

BACKGROUND AND TERMINOLOGY

Rotor response will be referenced herein as a function of load contribution by harmonic of the rotor angular speed, \( \Omega \), with \( iP \) denoting the \( i^{th} \)-per-rev contribution to the load (say, \( L(t) \), per the following Fourier expansion).

\[
L(t) = a_0 + \sum_{j=1}^{H} (c_j \cos j\Omega t + s_j \sin j\Omega t)
\]  

(1)

where \( H \) is the retained number of harmonics of the forcing frequency used in the expansion. Figure 2 shows the definition of rotor azimuth (\( \psi \)) as well as the relative positions of the blades for a 4-bladed rotor system (e.g., the UH-60A). As shown in Figure 3, rotor angular velocity, \( \Omega \), is defined as positive counter-clockwise, blade pitch (feathering) angle (\( \theta \)) is defined as positive blade leading edge up, blade flap angle (\( \beta \)) is defined as
positive blade tip up, and blade lag angle ($\zeta$) is defined as positive clockwise rotation (opposite the direction of blade angular rotation). Furthermore, let the blade torsional moment ($M_z$, or $ST$) be defined as positive blade leading edge up, blade flapwise bending moment ($M_x$, or $SN$; sometimes referred to as flatwise bending or normal bending) as positive blade tip up, blade edge-wise bending moment ($M_y$, or $SE$; sometimes referred to as lea-dlag bending) as positive counter-clockwise, and blade pushrod (pitch link) axial load ($F_z$, or $PL$) as positive in tension.

**Rotor Load Harmonic Composition**

Rotor control is primarily achieved by the placement of the swashplate. Swashplate position and orientation (driven by collective and cyclic control inputs, respectively) dictate the magnitude and direction of the rotor thrust vector, thus providing the mechanism through which the aircraft’s velocity vector (and, thus, aircraft motion) is defined. For an $n_b$-bladed rotor rotating at angular speed $\Omega$, in the local coordinate system of a given blade (blade 1, with the $x$-axis oriented along the blade span, positive outboard), the in-plane velocity at radial station $r$ (at blade azimuth $\psi$) can be idealized as follows [6].

$$V = \Omega r + V_\infty \sin \psi$$  \hspace{1cm} (2)

where $V_\infty$ is the forward speed of the aircraft. The lift due to the motion of the blade as a function of $\psi$ can be idealized as:

$$L(\psi) = \frac{1}{2} \rho S C_L(\psi) V^2(\psi)$$  \hspace{1cm} (3)

$C_L$ accounts for the increased local blade velocity on the advancing side and the reduction in local blade velocity on the retreating side. For a given blade, $C_L$ as a function of $\psi$ is:

$$C_L(\psi) = C_{L0} - \Delta C_L \sin \psi$$  \hspace{1cm} (4)

$C_{L0}$ is the lift at zero $\alpha$ and $\Delta C_L$ is some positive constant. The important concept is that $C_L$ reduces on the advancing side and increases on the retreating side to ensure comparable lift magnitudes for both; i.e., for steady, level flight, no resulting pitching or rolling moment:

$$L(0^\circ) = L(180^\circ); \ L(90^\circ) = L(270^\circ)$$  \hspace{1cm} (5)

Plugging Eqn. 2 and Eqn. 4 into Eqn. 3, then normalizing by the square of the blade tip speed ($\Omega r^2$) yields the following, upon trigonometric manipulation.

$$L = 0.5 \rho S (\Omega r)^2 \left\{ -\Delta C_L \left( \frac{r}{R} \right) \mu + C_{L0} \left( \left( \frac{r}{R} \right)^2 + 0.5 \mu^2 \right) \right\}$$

$$+ \left[ 2C_{L0} \left( \frac{r}{R} \right) \mu - \Delta C_L \left( \left( \frac{r}{R} \right)^2 + 0.75 \mu^2 \right) \right] \sin \psi$$

$$+ \left[ \Delta C_L \left( \frac{r}{R} \right) \mu - 0.5 C_{L0} \mu^2 \right] \cos(2\psi)$$

$$+ 0.5 \Delta C_L \mu^2 \sin(3\psi) \right\}$$  \hspace{1cm} (6)

The resulting expression of the lift contains $0P$, $1P$, $2P$, and $3P$ contributions. Assuming dominance in the response due to $\Delta C_L$, the $1P$ airload is proportional to $((r/R)^2 + 0.75 \mu^2)$, the $2P$ to $(r/R)\mu$, and the $3P$ to $0.5 \mu^2$. Thus, as one moves outboard on the blade, one expects the $1P$ airload to be much larger than the $2P$ and the $2P$, in turn, much larger than the $3P$ airload. Figure 4 shows experimentally-measured UH-60A blade normal force (per unit span) for radial station 0.225R and 0.775R versus $\mu$ for each flight counter of flight 85. This simplified representation of blade airloads does seem to hold true for the inboard station (0.225R), showing $1P > 2P > 3P$ response. However, this representation does not hold for airloads outboard on the blade. This is due to the unsteady airloads effects present outboard, such as negative lift on the advancing side, stall due to high blade angle-of-attack on the retreating sides (with higher $\mu$), and excitation of the first elastic flapping mode (2.87P, close to 3P).
FIGURE 4. UH-60A MEASURED BLADE NORMAL FORCE (PER UNIT LENGTH; LB/IN (N/M)) BY HARMONIC, V. ADVANCE RATIO (EACH FLIGHT COUNTER IN FLIGHT 85).

Main Rotor Shaft Bending

Next rotor shaft bending is examined. The rotor shaft is typically a fatigue critical component. The UH-60A dataset contains bending for an upper and a lower location on the shaft. The upper bending location (RQ12) is located below and centered with blade 2. Conceptually, this shaft bending due to blade lift should be equal to the net lift of blade 2 minus the net lift of blade 4 (positive bending in tension), multiplied by the appropriate moment arm (treating the blade lift as acting at some station $(x_sR)$ on the blade), which can be shown to be equal to:

$$M_{shaft} = \rho S x_R (\Omega R)^2 \left[ (-2C_{L0} x_s + \Delta C_L (x^2_s + 0.75\mu^2)) \cos \psi + 0.25\Delta C_L \mu^2 \cos(3\psi) \right] \tag{7}$$

This formulation contains $1P$ and $3P$ content, with $1P$ proportional to $(x^2_s + 0.75\mu^2)$ and $3P$ proportional to $0.25\mu^2$. Since the lift on a rotor blade is maximum outboard on the blade (say $x_s \approx 0.7$), the former term is much larger than the latter across typical operational UH-60A advance ratios ($\mu = 0$ to 0.4). Figure 5 shows experimentally-measured UH-60A shaft upper bending moment versus azimuth for each flight counter in flight 85. These data are consistent with the formulation presented in equation 7, dominated by $1P$ content, with minor $3P$ content.

The intent of the above examples was not to derive closed form solutions for rotor loads harmonic content, but, rather, to explain the fundamental physical principles behind higher harmonic content.

FIGURE 5. UH-60A MEASURED UPPER SHAFT BENDING (BENDING IN DIRECTION OF BLADE 2 FLAPPING; IN-LB (N-M)), BY HARMONIC, V. ADVANCE RATIO (EACH FLIGHT COUNTER IN FLIGHT 85).

LOAD TRANSFERENCE BETWEEN THE ROTATING AND FIXED FRAMES

A number of research efforts have focused on vibratory response and load transference between the fixed and rotating frames [7–10]. Traditionally, these efforts have addressed rotating loads prediction based on fixed-system measurements. This is a reasonable objective, given that fixed-system components are easier to instrument than rotating system components. This section will explain the challenges and limitations of these approaches.

The pushrod load $P$ (for a given blade) may be written via Fourier expansion as a function of rotor speed $\Omega$ as follows.

$$P(\psi) = P_0 + \sum_{j=1}^{H} (c_j \cos(j\psi) + s_j \sin(j\psi)) \tag{8}$$

Where azimuth is related to the time domain as $\psi = \Omega t$.

4-bladed Rotor

Figures 6 through 10 show the additive load shape of the pushrod loads for a 4-bladed rotor as a function of blade azimuth for five harmonics. From here it is easy to visualize the net effect on fixed system forces and moments. Figure 6 shows the relative position of the four pushrods’ $1P$ content $(c_1 \cos \psi + s_1 \sin \psi)$ throughout a single rotation at rate $\Omega$. At any given position, the sum of the vertical forces due to the four pushrods is zero. For non-zero longitudinal $(s_1)$ and lateral $(c_1)$ pushrod response, $1P$ bending moments are transferred to the fixed frame. The same holds true for $5P$ (Figure 10), $9P$, ..., $(jn_0 + 1)P$. This is known as a cyclic mode - or whirl mode, in the context of harmonics above $1P$. Cyclic/whirl modes can be thought of as modes than transfer moments but zero forces.

Figure 7 shows the relative position of the four pushrods’ $2P$ content $(c_2 \cos(2\psi) + s_2 \sin(2\psi))$ throughout a single rotation at rate $\Omega$. At any given position, both the sum of the vertical forces...
and the sum of the bending moments are zero. The same holds true for $6P, 10P, \ldots (jn_b + 2)P$. This is known as a reactionless (or warp) mode. An interesting observation of the reactionless mode is that any attempt at using fixed system components to estimate the pushrod load will be unable to capture its $2P$ content, which can be a non-negligible component (Figure 14). Reactionless modes can be thought of as modes with zero force or moment transfer.

Figure 8 shows the relative position of the four pushrods' $3P$ content $(c_3 \cos(3\psi) + s_3 \sin(3\psi))$ throughout a single rotation at rate $\Omega$. At any given position, the sum of the vertical forces is zero. For non-zero longitudinal $(s_3)$ and lateral $(c_3)$ pushrod response, $3P$ bending moments are transferred to the fixed frame. It has the same net effect as $1P, 5P, \ldots$,

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Thus, in the context of this idealization of pushrod response, only \( j n_b P \) force components \((j = 0, 1, 2, \ldots)\) are fed from the rotating frame into the fixed frame, where \( n_b \) is the number of blades. Note that the above idealization ignores swashplate dynamics (discussed below).

Alternatively, it could be phrased that the sum of all pushrod loads is composed of \( j n_b P \) force components’ \((j = 0, 1, 2, \ldots)\) content (e.g., \( 0P + 4P + 8P + \ldots \)). This is shown in Figure 12, where pushrod and control servo load FFTs are plotted vs. advance ratio for flight 84. Note the equivalence between the summed servo loads and the summed pushrod loads. This is also shown in Figure 9, where the four pushrod loads additively produce a non-zero net \( F_z \) at \( 4P \) (with zero \( F_z \) at \( 1P, 2P, 3P, 5P \), etc.; see Figures 6, 7, 8, and 10, respectively).

Now moments are examined. From Figure 11, the moments summed about \( x_s \) and \( y_s \) should each be zero, since no external moments are applied. The moment summed about \( x_s \) is then written as:

\[
(LS)R_s + (P_3 - P_1)R_p \cos(\psi - \psi_{0s}) + (P_4 - P_2)R_p \sin(\psi - \psi_{0s}) = 0 \quad (12)
\]

The moment summed about \( y_s \) is then written as:

\[
(FS - AS)R_s - (P_3 - P_1)R_p \sin(\psi - \psi_{0s}) + (P_4 - P_2)R_p \cos(\psi - \psi_{0s}) = 0 \quad (13)
\]

These two moment equations are of interest in that they each difference opposing pushrod loads \((P_3 - P_1) \text{ and } P_4 - P_2\). These two differenced terms are expanded as follows.

\[
(P_3 - P_1) = \varepsilon_3 - 2 \sum_{j=1,3,5,\ldots}^{H} [c_j \cos(j\psi) + s_j \sin(j\psi)] \quad (14)
\]

\[
(P_4 - P_2) = \varepsilon_4 - \varepsilon_2 - 2 \sum_{j=1,3,5,\ldots}^{H} \left\{ (-1)^{\frac{j+1}{2}} [c_j \sin(j\psi) - s_j \cos(j\psi)] \right\} \quad (15)
\]

This shows that the differenced opposing pushrod loads \((P_3 - P_1) \text{ and } P_4 - P_2\) are driven by odd harmonics \((1P, 3P, 5P, \text{ etc.})\). This is verified with UH-60A flight test data in Figure 12.
dominates, but there is relevant content at $3P$ and, to a lesser degree, $5P$ as well. This was demonstrated visually in Figures 6 through 10. Opposing pushrod loads have the same sign for even harmonics but opposite sign for odd harmonics (same magnitude in both cases), thus differencing opposing pushrod loads cancels the even harmonic effects but adds the odd harmonic effects.

Plugging Eqn. 14 and Eqn. 15 into Eqn. 12 yields the closed form solution for the lateral servo load, $LS$:

$$LS = \frac{R_p}{R_s} \left\{ 2c_1 \cos \psi_0 + 2s_1 \sin \psi_0 - \varepsilon_3 \cos (\psi - \psi_0) + (\varepsilon_2 - \varepsilon_4) \sin (\psi - \psi_0) + 2 \sum_{j=4,8,12,...}^H \left[ c_{j-1} \cos (j\psi - \psi_0) + c_{j+1} \cos (j\psi + \psi_0) + s_{j-1} \sin (j\psi - \psi_0) + s_{j+1} \sin (j\psi + \psi_0) \right] \right\}$$  (16)

Now Eqn. 11, Eqn. 13, and Eqn. 16 can be used to solve for the forward servo (FS) and aft servo (AS) loads, as shown in Eqn. 17 and Eqn. 18, respectively. These results (Eqn. 16, Eqn. 17, and Eqn. 18) show that the control servo loads in the fixed system have a zero frequency term combined with dominant effects due to $4P$ content. The $1P$ content is present as a function of the noise in the system ($\varepsilon_k$), causing zero frequency offset (i.e., bias) between the four pushrod loads. Figure 13 shows the actual frequency content of the control servo loads. As shown, $4P$ is dominant, with some observable $1P$ content, along with $2P$ and $3P$ contributions, close in magnitude to the $1P$ response. This actual $2P$ and $3P$ content is due to this idealization’s incorrect assumption that pushrods 2 and 4 would share identical $2P$ content, as would pushrods 1 and 3.

Figure 14 shows a time history and FFT for the four pushrod loads for flight counter c8534 ($\mu = 0.368$). As shown, there is some difference in magnitude, phasing, as well as frequency content between pushrods. Kufeld et al. [3] attribute some of the differences seen between pushrods at higher harmonics to blade-to-blade differences. However, given the dominance of $4P$ in the control servo loads, the above idealization is adequate for the goal of conceptual understanding of fixed and rotating system harmonic content addressed herein.

The next step is to use Eqn. 16, Eqn. 17, and Eqn. 18 to formulate control servo loads based on measured UH-60A pushrod
loads for blades 1 through 4 and compare to the measured control servo loads. Figure 15 shows this comparison for flight counter c8418 ($\mu = 0.094$). There is an under-prediction of 2P, 3P, and 4P for FS, an over-prediction of 2P and 4P for AS, and an under-prediction in LS. Overall, however, a reasonable approximation is obtained. The relevance of this is as follows. If the objective was to model pushrod response based on measured servo loads, it would be impossible to predict any pushrod loads’ 2P content (for a 4-bladed rotor) based on these measured servo loads. Additionally, this would lead to an under-determined system, given that there are four unknowns, i.e., the four pushrod loads, but only three equations: $\sum F_c$, $\sum M_{cs}$, and $\sum M_{vs}$.

\[
\begin{align*}
AS &= -2P_0 - \frac{1}{2} \sum_{k=2}^{4} \epsilon_k \\
&+ \frac{R_p}{R_s} [-c_1 (\cos \psi_0 + \sin \psi_0) + s_1 (\cos \psi_0 - \sin \psi_0)] \\
&+ \frac{1}{2} \frac{R_p}{R_s} [(-\epsilon_2 + \epsilon_3 + \epsilon_4) \cos (\psi - \psi_0) - (\epsilon_2 + \epsilon_3 - \epsilon_4) \sin (\psi - \psi_0)] \\
&- 2 \sum_{j=4,8,12,\ldots}^H [c_j \cos (j\psi) + s_j \sin (j\psi)] \\
&- \sum_{j=4,8,12,\ldots}^H \{c_{j-1}[(\cos (j\psi) - \psi_0) - (j\psi - \psi_0)] \\
&+ c_{j+1}[(\cos (j\psi + \psi_0) - \psi_0) + (j\psi + \psi_0)] \\
&+ s_{j-1}[(\cos (j\psi - \psi_0) + \psi_0) - (j\psi - \psi_0)] \\
&- s_{j+1}[(\cos (j\psi + \psi_0) + \psi_0) + (j\psi + \psi_0)]\}
\end{align*}
\] (17)

Swashplate Dynamics

Research by Abhishek et al. [12] has shown that contributions of the non-zero swashplate mass ($m_{sp},a_{sp}$ added to the RHS of Eqn. 10) can have appreciable effect on 4/rev servo loads (up to 25% variation in peak-to-peak load) but minimal effect on blade loads. These effects were not included herein. They could, perhaps, describe some of the differences seen in Figure 15.

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7-bladed Rotor

Figures 16 through 19 show the additive load shape of the pushrod loads for a 7-bladed rotor as a function of blade azimuth for 7 harmonics. From here it is easy to visualize the net effect on fixed system forces and moments. Figure 16 shows the relative position of the seven pushrods’ 1P content \((c_1 \cos \psi + s_1 \sin \psi)\) throughout a single rotation at angular speed \(\Omega\). At any given position, the sum of the vertical forces is zero. For non-zero longitudinal \((s_1)\) and lateral \((c_1)\) pushrod response, 1P bending moments are transferred to the fixed frame. The same holds true for 8P, 15P, ... \((jn_b+1)P\). This is known as a cyclic mode.

Figure 17 shows the relative position of the seven pushrods’ 2P content \((c_2 \cos(2\psi) + s_2 \sin(2\psi))\) throughout a single rotation at rate \(2\Omega\). At any given position, the sum of the vertical forces is zero. For non-zero longitudinal \((s_2)\) and lateral \((c_2)\) pushrod response, 2P bending moments are transferred to the fixed frame. The same holds true for 3P through 5P (not shown) as well as 9P through 12P. These are known as reactionless/warp modes.

Figure 18 shows the relative position of the seven pushrods’ 6P content \((c_6 \cos(2\psi) + s_6 \sin(2\psi))\) throughout a single rotation at rate \(6\Omega\). At any given position, the sum of the vertical forces is zero. For non-zero longitudinal \((s_6)\) and lateral \((c_6)\) pushrod response, 6P bending moments are transferred to the fixed frame. The same holds true for 13P, 20P, ... \((jn_b+1)P\). This is known as a whirl mode.

Figure 19 shows the relative position of the seven pushrods’ 7P content \((c_7 \cos(7\psi) + s_7 \sin(7\psi))\) throughout a single rotation at rate \(7\Omega\). At any given position, the sum of the bending moments is zero, but the sum of the vertical forces can be non-zero, with all pushrod 7P components acting in the same direction. The same holds true for 14P, 21P, ... \((jn_b)P\). This is a collective mode.

SUMMARY

This paper examined rotor load content by harmonic. For a 4-bladed rotor system, blade lift due to simplified aerodynamics was derived and compared with measured UH-60A results. Inboard response of the blade followed the experimental trends. Outboard blade response did not, due to unsteady airloads effects.

The mechanics of load transference between the rotating and fixed frames were also examined. For the UH-60A, it was shown how the sum of all pushrod loads is composed of \(jn_bP\) force components \((j = 0, 1, 2, \ldots)\) content (e.g., \(0P + 4P + 8P + \ldots\)). Closed form expressions for the three UH-60A servo loads were derived as well. It was shown how, if the objective was to model pushrod response based on measured servo loads, it would be impossible to predict any pushrod loads’ 2P content (for a 4-bladed rotor) based on these measured servo loads.

A generic 7-bladed rotor was also examined. For both the 4- and 7-bladed rotors, cyclic/whirl, reactionless/warp, and collective modes were defined and illustrated. The following generalizations may be applied to an \(n_b\)-bladed rotor (where \(j = 1, 2, \ldots\)):

1. \((jn_b-1)\), and \((jn_b+1)\) are rotor cyclic/whirl modes
2. \(jn_b\) are collective modes
3. All others are reactionless/warp modes

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REFERENCES

FIGURE 16. 7-BLADED ROTOR FIXED SWASHPLATE FORCES/MOMENTS (AS FN OF $\psi$) DUE TO PUSHROD AXIAL LOADS: 1P HARMONIC (CYCLIC MODE). NET RESULT: ZERO $\Sigma F_z$; NON-ZERO $\Sigma M_y$ (LEFT), $\Sigma M_x$ (RIGHT).

FIGURE 17. 7-BLADED ROTOR FIXED SWASHPLATE FORCES/MOMENTS (AS FN OF $\psi$) DUE TO PUSHROD AXIAL LOADS: 2P HARMONIC (REACTIONLESS/WARP MODE). NET RESULT: NON-ZERO $\Sigma F_z$; ZERO $\Sigma M_y$ (LEFT), $\Sigma M_x$ (RIGHT).

FIGURE 18. 7-BLADED ROTOR FIXED SWASHPLATE FORCES/MOMENTS (AS FN OF $\psi$) DUE TO PUSHROD AXIAL LOADS: 6P HARMONIC (CYCLIC/WHIRL MODE). NET RESULT: ZERO $\Sigma F_z$; NON-ZERO $\Sigma M_y$ (LEFT), $\Sigma M_x$ (RIGHT).

FIGURE 19. 7-BLADED ROTOR FIXED SWASHPLATE FORCES/MOMENTS (AS FN OF $\psi$) DUE TO PUSHROD AXIAL LOADS: 7P HARMONIC (COLLECTIVE MODE). NET RESULT: NON-ZERO $\Sigma F_z$; ZERO $\Sigma M_y$ (LEFT), $\Sigma M_x$ (RIGHT).


